

Transport properties of a charged drop in an external electric field

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Abstract

Transport properties of a charged droplet of weakly interacting particles in an external field are investigated. A non-equilibrium distribution function which describes a process of the droplet transverse evolution with constant entropy in an external electric field is calculated. With the help of this distribution function, shear viscosity coefficients in the transverse plane are calculated as well. They are found to be very small and depend on the time of the droplet's expansion in a hydrodynamical regime and external field value. An applicability of the results to the description of initial states of quark-gluon plasma obtained in high-energy interactions of nuclei is also discussed.

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1 Introduction

Collisions of relativistic nuclei in the RHIC and LHC experiments at very high energies led to the revelation of a new state of matter named quark-gluon plasma (QGP). At the initial stages of the scattering, this plasma is assumed to be in a strongly interacting phase and resembles as a liquid referred as a strongly coupled Quark Gluon Plasma (sQGP) [1], whose microscopic structure is not well understood yet [1, 2, 3, 4, 5, 6, 7, 8, 9]. Anyway, the data obtained at the RHIC experiments are in a good agreement with the predictions of the ideal relativistic fluid dynamics, [10, 11], which establish fluid dynamics as a main theoretical tool to describe the collective flow in the collisions. As an input to the hydrodynamical evolution of the particles it is assumed that after a very short time, $\tau < 1 \text{ fm/s}$ [12], the matter reaches a thermal equilibrium and expands with a very small shear viscosity [13, 14].

In the process of the high-energy scattering, the thermal equilibrium may be achieved only for small fireballs [15] of the matter [3, 8, 14, 16]; the whole colliding system cannot be in a global equilibrium state because the nuclei scattering at high energy is a highly non-equilibrium process [3, 17]. Subsequent expansion of the matter's hot spot occurs with the constant entropy [18], which justifies the applicability of the hydrodynamical description of the process. This adiabatic expansion continues till the value of the particle's mean free path becomes comparable with the size of the system. In this stage, instead a liquid, a gas of interacting particles whose density rapidly decreases reveals.

Application of the fluid dynamics to the process of the fireball's expansion requires some initial conditions among which the most intriguing one is a small value of the shear viscosity/entropy ratio. Perturbative result for the shear viscosity calculations is large, [19], and some new mechanisms of the explanation of the shear viscosity smallness are required. There are different approaches to the possible mechanisms of this smallness. Except the models of strongly interacting quarks and gluons [1], there are approaches based on the weakly interacting particles with novel mechanism of the viscosity creation, see for example [20, 21, 22]. In this note we also propose some new mechanism responsible for the shear viscosity smallness, similar in some extent to the ideas of [21, 22]. Namely, we consider a model of a small and very dense charged droplet, see [25], which inter-particle interactions are weak similarly to the interactions of the asymptotically free quarks and gluons in QCD. We investigate a hydrodynamical expansion of this unstable droplet and calculate a viscosity of the process during the collisionless regime of the expansion. Following to the [21] we also call obtained viscosity as anomalous one, despite the fact that the mechanism of the viscosity coefficients smallness in our approach is different from the proposed in [21].

In our calculations we consider only an transverse expansion of small, [25], and dense droplet of charged particles. Following by [18], we consider the process of this expansion as the process with constant entropy. Due to the fact that we consider a charged droplet, the distribution function of the system cannot be stationary, it must depend on time. Thereby, the expansion process of the charged droplet with the constant entropy is determined by Vlasov's equation, [24], which is the main tool for the kinetic description of the process. We solve the Vlasov's equation for the time-dependent, non-equilibrium distribution function which being time-dependent anyway preserves constant value of the entropy. Moreover, we consider our hot spot in the transverse external field of the other, relativistic particles, see [25], which contributes to the Vlasov's equation solution. As we will see further, the interaction between the charged droplet and external field is the mechanism responsible for the small values of the shear viscosity coefficients in the given framework.

In the next Section 2, we derive the electromagnetic field potentials created by the relativistically moved charged drop. In Section 3, we write the Vlasov's equation for the charged drop in the external electric field, whereas in Section 4, we rewrite the Vlasov's equation in a new, integral form. Also, in Section 4, we determine a new initial condition for the equation which is different from the given in Appendix A because of the presence of the external electric field. In further Section, Section 5, we calculate a non-equilibrium distribution function till the first order of the perturbative series, which is formulated and defined in this Section. In the Section 6 we investigate transport characteristic of the charged droplet in the external electric field, including transverse shear viscosity coefficients. The last Section 7 is a conclusion of the paper.

2 An external field of relativistic charged drop

To a first approximation, we consider a charged drop of the matter moving as whole with some velocity. In the frame related with this drop, the distribution function of the droplet's particles is described by usual Vlasov's equations:

$$\frac{\partial f_s}{\partial t} + \vec{v} \frac{\partial f_s}{\partial \vec{r}} + \left(q\vec{E} + \frac{q}{c} \vec{v} \times \vec{B} \right) \frac{\partial f_s}{\partial \vec{p}} = 0. \quad (1)$$

with Maxwell's equations

$$\nabla \times \vec{E} = 0, \quad (2)$$

$$\nabla \times \vec{B} = 0, \quad (3)$$

$$\nabla \cdot \vec{E} = 4\pi q n \int f_s(x, t) d^3v, \quad (4)$$

$$\nabla \cdot \vec{B} = 0. \quad (5)$$

In this approximation, for the usual Maxwellian initial distribution function, the solution for the self-consistent field is the screened electrostatic potential plus some constant which can be considered further as an initial value of the potential:

$$\varphi = C_0 \frac{e^{-r/r_D}}{r} + 4\pi q n r_D^2 \quad (6)$$

see [25, 26] for example. Here C_0 is some constant and

$$r_D^2 = \frac{k_B T_0}{4\pi q^2 n_0} \quad (7)$$

is Debye length. Redefining the potential's initial value we obtain finally the following expression for the potential

$$\varphi = C_0 \frac{e^{-r/r_D}}{r} \quad (8)$$

which formally can be regarded as originating from the exchange of "massive photons" with masses $M = 1/r_D$.

Now, let's consider the field of Eq. (8) in the rest frame of some another dense drop of particles, which is in the rest relatively to the first drop. In this frame, the potential Eq. (8) is created by fast moving dense cloud of the particles with the volume V_0 , particle's density n and total charge $q n V_0$. In this reference frame, therefore, the potential Eq. (8) is described by Proca equation which has the following form:

$$\Delta \varphi - \frac{1}{c^2} \frac{\partial^2 \varphi}{\partial t^2} - \frac{\varphi}{r_D^2} = -4\pi q n_0 V_0 \delta(\vec{r} - \vec{r}_s(t)) \quad (9)$$

Solution of this equation is well known:

$$\varphi(\vec{r}, t) = \frac{q n_0 V_0}{2\pi^2} \int d^2 k_\perp e^{ik_\perp(r_\perp - b)} \int_{-\infty}^{\infty} \frac{e^{ik_z(z - vt)} dk_z}{k_\perp^2 + k_z^2 \gamma + 1/r_D^2} \quad (10)$$

where $\vec{r} = (r_\perp, z)$, the position of the moving drop is given by $\vec{r}_s = (b, vt)$ with $r_\perp = (r_x, r_y)$, $b = (b_x, b_y)$ and $\gamma = \sqrt{1 - \frac{v^2}{c^2}}$. In the relativistic limit when $v \approx c$ we obtain in the first order expansion over γ for the electrostatic potential:

$$\varphi(\vec{r}, t) = 2q n_0 V_0 \delta(z - ct) K_0(|r_\perp - b|/r_D) \quad (11)$$

where K_0 is Macdonald's function [30]. Correspondingly, for the vector potential of the electromagnetic field we have:

$$A_z(\vec{r}, t) = \varphi(\vec{r}, t), \quad (12)$$

$$A_x(\vec{r}, t) = A_y(\vec{r}, t) = 0. \quad (13)$$

After the gauge transformation with the gauge function

$$f = -2q n_0 V_0 \theta(z - ct) K_0(|r_\perp - b|/r_D) \quad (14)$$

we finally obtain the potentials of the moving drop in the rest frame of another dense droplet:

$$\varphi(\vec{r}, t) = A_z(\vec{r}, t) = 0, \quad (15)$$

$$A_x(\vec{r}, t) = -2q n_0 V_0 \theta(z - ct) \partial_x K_0(|r_\perp - b|/r_D), \quad (16)$$

$$A_y(\vec{r}, t) = -2q n_0 V_0 \theta(z - ct) \partial_y K_0(|r_\perp - b|/r_D). \quad (17)$$

Here θ is the Heaviside step function and coordinate dependence of the potentials is factorized by two parts: the θ function depends only on the longitudinal z coordinate whereas the second function depends on the transverse x and y coordinates. There is a factorization of the dimensions. Therefore, further, we will consider only two dimensional dynamics of the droplet, taking $v_z = 0$ for the droplet in the rest.

3 Vlasov's equation of the charged droplet in the external field

In this section, we consider an interaction between two different charged spots, one of which was created at the first stage of the interaction and stays in the rest and the second is moving towards the first one. Again, both droplets we consider as "rigid" drops, which interact at some initial moment of time. After the interaction, the second drop begins its expansion/compression in the transverse plain. We limit the dynamics in two dimensional plain, taking $v_z = 0$ for the expanding/compressing droplet. This Vlasov's description of the droplet's dynamics is valid during some time T , after which the collision processes between particles inside the droplet became to be important. The possible value of T we will estimate below.

The system of equations Eq. (1), Eq. (2) may be reformulated as a system of self-consistent field of the first spot in the field of the second. In this case instead Eq. (1) we have:

$$\frac{\partial f_s}{\partial t} + \vec{v} \frac{\partial f_s}{\partial \vec{r}} + q \vec{E}_{total} \frac{\partial f_s}{\partial \vec{p}} = 0. \quad (18)$$

Here we neglected magnetic field. The reason for that is simple. The potentials Eq. (15)-Eq. (17) describe magnetic field with only non-zero azimuthal component:

$$B_\theta = \partial_z A_r, \quad (19)$$

here $A_r(\vec{r}, t)$ is a radial component of the potential, see Eq. (24) below. The same is correct for the self-consistent magnetic field of the droplet. Therefore, because in the 2-dimensional, transverse Vlasov's equation the main magnetic field contribution in the force component has a form $v_z B_\theta / c$, when $v_z = 0$, we obtain that only transverse electric field remains in the model. In general it means, that in calculations we will neglect the radial, so called self-focusing, component of the force in three dimensions. We argue in the conclusion that it does not affect on the main results of the manuscript.

Thereby, in Eq. (18) we have:

$$\vec{E}_{total} = \vec{E}_s - \vec{E}_{ext} \quad (20)$$

with \vec{E}_s as a new self-consistent field and \vec{E}_{ext} as a field of the second, incident drop. Maxwell's equations for the self-consistent field in this case have a following form

$$\nabla \times \vec{E}_s = 0, \quad (21)$$

$$\nabla \cdot \vec{E}_s = 4\pi q n \int f_s(x, t) d^3v. \quad (22)$$

The vector potential which determines the \vec{E}_{ext} field can be found by using Eq. (16)-Eq. (17) from the previous section. In cylindrical coordinates with the origin in the center of the rest spot, where we take $r_\perp = r$, we have:

$$\vec{E}_{ext} = -\frac{1}{c} \frac{\partial A_r}{\partial t} \hat{e}_r \quad (23)$$

with the field

$$A_r(\vec{r}, t) = A_x(\vec{r}, t) \cos \theta + A_y(\vec{r}, t) \sin \theta = -2q n_0 V_0 \theta(z - ct) \frac{\partial K_0(|r - b|/r_{0D})}{\partial r}. \quad (24)$$

Defining an overall charge of the incident drop equal to

$$Q_T = q n_0 V_0, \quad (25)$$

we have:

$$A_r(\vec{r}, t) = -2Q_T \theta(\tau) \frac{\partial K_0(|r - b|/r_{0D})}{\partial r}. \quad (26)$$

Here we denoted $\tau = ct$ because of the new rest frame with the origin in the droplet in rest, therefore $\tau > 0$ determines the evolution of the drop after the interaction with external field at $\tau = 0$.

Also, denoting $\xi_0 = \frac{r}{r_{0D}}$, we obtain :

$$A_r(\vec{r}, t) = -2 \frac{Q_T}{r_{0D}} \frac{\partial K_0(\xi_0)}{\partial \xi_0} \theta(\tau) = F(r) \theta(\tau). \quad (27)$$

For the c.m.f. related with the droplet in the rest, we have $r = 0$ and $b \rightarrow r$, so introducing a cut-off around $r = 0$ we denote in the following $K_0(\xi_0) = K_0(|r - r_{0D}|/r_{0D})$. We underline, that the total charge of the incident drop Q_T depends in general on a total energy of the process in the c.m.f. .

For the Vlasov's equation Eq. (18), redefining as well the time variable as $t \rightarrow ct = \tau$, we obtain:

$$c \frac{\partial f_s}{\partial \tau} + \vec{v} \frac{\partial f_s}{\partial \vec{r}} + q \left(\vec{E}_s - \vec{E}_{ext} \right) \frac{\partial f_s}{\partial \vec{p}} = 0. \quad (28)$$

with

$$\vec{E}_{ext} = \frac{\partial A_r}{\partial \tau} \hat{e}_r = F(r, s) \delta(\tau) \hat{e}_r, \quad (29)$$

where $F(r, s)$ is the function from Eq. (26). In Eq. (28) there is no z dependence neither in the distribution function nor in the electric field. The vectors \vec{r}, \vec{p} are radial there, therefore Eq. (28) describes an evolution of the distribution function in two dimensional transverse plane as a function of the "time" parameter τ . Also, we see, that because our distribution function does not depend on the azimuthal angle and because the external field is purely radial, the equation Eq. (28) in fact is one dimension with only radial dependence included.

4 Integral form of Vlasov equation and new initial conditions

In order to analyze an analytical solution of Eq. (28)-Eq. (29) it is more convenient to rewrite Vlasov's equation as an integral one. First of all, we rewrite this equation as following:¹:

$$\frac{1}{\zeta_r} \frac{\partial f_s}{\partial \tau} + \frac{\partial f_s}{\partial r} = -\frac{q}{c \zeta_r} (E_{rs} - E_{rext}) \frac{\partial f_s}{\partial p_r} \quad (30)$$

with $\zeta_r = v_r/c^2$. For the l.h.s. of this equation, we find the fundamental solution from the following equation:

$$\frac{1}{\zeta_r} \frac{\partial f_s}{\partial \tau} + \frac{\partial f_s}{\partial r} = \delta(\tau) \delta(r), \quad (31)$$

this solution is well known, see for example [31], it is given by

$$\mathcal{E} = \zeta_r \theta(\tau) \delta(r - \zeta_r \tau). \quad (32)$$

Thereby, with the help of Eq. (32), we rewrite Eq. (30) as an integral equation:

$$f_s(r, \vec{\zeta}, \tau) = -\frac{q}{c \zeta_r} \int dr' d\tau' \mathcal{E}(\tau - \tau', r - r') \left(E_{rs}(r', \tau') - E_{rext}(r', \tau') \right) \frac{\partial f_s(r', \vec{\zeta}, \tau')}{\partial p_r(\zeta_r)} + f_0(r - \zeta_r \tau, \vec{\zeta}), \quad (33)$$

¹In our problem a velocity on the z axis is decoupled from the radial and angle velocities, i.e. $f_s \propto G(v_z) f_s(v_\theta, v_r)$ with $\int G(v_z) dv_z = 1$,

²In the following we will denote $\vec{\zeta} = (\vec{\zeta}, \zeta_\theta) = (v_r/c, v_\theta/c)$.

where the function $f_0(r, \vec{\zeta})$ will be determined later. Inserting Eq. (32) into Eq. (33), we obtain³ :

$$\begin{aligned} f_s(r, \vec{\zeta}, \tau) &= \\ &= -\frac{q}{c} \int_0^\tau d\tau' \left(E_{rs}(r - \zeta_r(\tau - \tau'), \tau') - E_{rext}(r - \zeta_r(\tau - \tau'), \tau') \right) \frac{\partial f_s(r - \zeta_r(\tau - \tau'), \vec{\zeta}, \tau')}{\partial p_r(\zeta_r)} + \\ &+ f_0(r - \zeta_r \tau, \vec{\zeta}). \end{aligned} \quad (34)$$

With the help of expression for the self-consistent field

$$E_{rs}(r, \tau) = \frac{4\pi q n}{r} \int dz z \int f_s(z, v_r, v_\theta, \tau) d^2v, \quad (35)$$

and for the external field

$$E_{rext}(r, \tau) = -2 \frac{Q_T}{r_{0D}} \frac{\partial K_0(\xi_0)}{\partial \xi_0} \delta(\tau) = 2 \frac{Q_T}{r_{0D}} K_1(\xi_0) \delta(\tau), \quad (36)$$

we obtain for Eq. (34):

$$\begin{aligned} f_s(r, \vec{\zeta}, \tau) &= -\frac{4\pi q^2 n}{rc} \int_0^\tau d\tau' \frac{\partial f_s(r - \zeta_r(\tau - \tau'), \vec{\zeta}, \tau')}{\partial p_r(\zeta_r)} \int^{r - \zeta_r(\tau - \tau')} dz z \int f_s(z, v'_r, v'_\theta, \tau') d^2v' + \\ &+ \frac{2q Q_T}{c r_{0D}} K_1(\xi_0 - \zeta_r \frac{\tau}{r_{0D}}) \frac{\partial f_{s0}(r - \zeta_r \tau, \vec{\zeta})}{\partial p_r(\zeta_r)} + f_0(r - \zeta_r \tau, \vec{\zeta}). \end{aligned} \quad (37)$$

At $\tau = 0$ this equation gives an equation for the f_{s0} :

$$f_{s0}(r, p_r) - \frac{2q Q_T}{c r_{0D}} K_1(\xi_0) \frac{\partial f_{s0}(r, p_r)}{\partial p_r(\zeta_r)} = f_0(r, p_r) \quad (38)$$

with some initial function f_0 . Therefore, Eq. (37) can be written in simpler form:

$$\begin{aligned} f_s(r, \vec{\zeta}, \tau) &= -\frac{4\pi q^2 n}{rc} \int_0^\tau d\tau' \frac{\partial f_s(r - \zeta_r(\tau - \tau'), \vec{\zeta}, \tau')}{\partial p_r(\zeta_r)} \cdot \\ &\cdot \int^{r - \zeta_r(\tau - \tau')} dz z \int f_s(z, v'_r, v'_\theta, \tau') d^2v' + f_{s0}(r - \zeta_r \tau, \vec{\zeta}). \end{aligned} \quad (39)$$

Equations Eq. (38)-Eq. (39) are full analogues of the initial Vlasov equations with some initial condition defined by Eq. (38).

An analytical solution of the Eq. (38) can be easily found. For the case of non-relativistic radial momenta, which is a case of interest and when $v_r/c < 1$, it is

$$f_{s0}(r, \zeta_r) = \frac{1}{\Lambda_0 K_1(\xi_0)} e^{\zeta_r/(\Lambda_0 K_1(\xi_0))} \int_{\zeta_r}^\infty f_0(r, \zeta'_r) e^{-\zeta'_r/(\Lambda_0 K_1(\xi_0))} d\zeta'_r, \quad (40)$$

where

$$\Lambda_0 = \frac{2q Q_T}{mc^2 r_{0D}} \approx \frac{E_p^{ext}}{\mathcal{E}_{kin}} \quad (41)$$

is the parameter which depends only on the field of the incident drop. Here E_p^{ext} is a potential energy of the interaction of the particle with the incident drop,

$$E_p^{ext} \propto \frac{q Q_T}{r_{0D}}, \quad (42)$$

and \mathcal{E}_{kin} is the relativistic kinetic energy of the incident drop. Therefore, there are two different regimes in our problem, the high-energy (relativistic or weak external field limit) one when $\Lambda_0 < 1$ and another one of the strong external field when $\Lambda_0 > 1$. Solution Eq. (40) is valid in both cases

³Here and in the following expressions firstly the derivative over $p_r(\zeta_r)$ is taken and after the value of $r - \zeta_r(\tau - \tau')$ is inserting in the expression for distribution function.

but we are interesting in the high-energy regime when $\Lambda_0 < 1$ as $s \rightarrow \infty$ ⁴. In this case, we search our function in the form of the following series:

$$f_{s0}(r, \vec{\zeta}) = \sum_{i=0}^{\infty} F_i^{s0} \Lambda_0^i. \quad (43)$$

From Eq. (38) or from Eq. (40), in the first two orders of the approximation, we obtain:

$$f_{s0}(r, \vec{\zeta}) = f_0(r, \vec{\zeta}) + \Lambda_0 K_1(\xi_0) \frac{\partial f_0(r, \vec{\zeta})}{\partial \zeta_r}. \quad (44)$$

This form is useful for the case of high-energy (relativistic) limit of the problem for any form of the initial distribution function $f_0(r, p_r)$.

Coming back to our initial differential formulation of Vlasov equation Eq. (28), we see that Eq. (39) is equivalent to the usual Vlasov equation:

$$c \frac{\partial f_s}{\partial \tau} + v_r \frac{\partial f_s}{\partial r} + q E_{rs} \frac{\partial f_s}{\partial p_r} = 0 \quad (45)$$

with only self-consistent field included. The only difference of the new equation from the usual formulation is the initial conditions, Eq. (38), where the influence of the external field is included. Somehow it is very predictable result. Because of its factorized form, the external field may be considered as an additional source of the perturbation which acts only at initial time of the process of drop's compression/expansion by the external field.

5 Solution of Vlasov's equation for "rigid-body" initial equilibrium distribution.

5.1 Initial condition and form of the perturbative expansion

For the further calculations of the averaged values of the parameters of the problem. we need to solve Vlasov's equation and calculate distribution function. We consider the integral equation Eq. (39), obtained above, in the case $\Lambda_0 < 1$. Solution of our initial Vlasov's equation, therefore, is given by Eq. (28). Considering this equation as perturbative one, with "time" τ as a small parameter, we write:

$$f_s(r, \vec{\zeta}, \tau) = \sum_{i=0}^{\infty} f_{si}(r, \vec{\zeta}) \tau^i. \quad (46)$$

In the first order on τ we have

$$f_s(r, \vec{\zeta}, \tau = 0) = f_{s0}(r, \vec{\zeta}) = f_0(r, \vec{\zeta}) + \Lambda_0 K_1(\xi_0) \frac{\partial f_0(r, \vec{\zeta})}{\partial \zeta_r}, \quad (47)$$

see Eq. (44). The initial function $f_0(r, \vec{\zeta})$ we choose as a rotating "rigid-body" equilibrium distribution function described in Appendix A. The reason for this choice is that this form of the distribution function is mostly appropriate for the configuration of the fields created in the high-energy scattering, see [32].

5.2 First order term of the distribution function

In order to clarify a solution of our equation Eq. (39), we perform Fourier transform over r variable of our functions of interest:

$$f_s(r, \vec{\zeta}, \tau) = \int e^{ikr} f_{sk}(\vec{\zeta}, \tau) dk. \quad (48)$$

⁴Here s is a squared total energy of the scattering process in the drop's rest frame.

Taking into account the normalization of the initial distribution function,

$$\int d^2 v f_{s0}(r, \vec{v}) = 1, \quad (49)$$

see Eq. (A.14), the equation Eq. (39), therefore, acquires the following form:

$$\begin{aligned} & \int e^{ikr} \left(f_{sk0}(\vec{\zeta}) + f_{sk1}(\vec{\zeta}) \tau \right) dk = \\ & = -\Lambda \int_0^\tau \left(r - \zeta_r \tau' \right)^2 d\tau' \frac{\partial}{\partial \zeta_r} \int e^{ik(r - \zeta_r \tau')} f_{sk0}(\vec{\zeta}) dk + \int e^{ik(r - \zeta_r \tau)} f_{sk0}(\vec{\zeta}) dk, \end{aligned} \quad (50)$$

with

$$\Lambda = \frac{2\pi q^2 n}{rmc^2}. \quad (51)$$

Thereby, in the first order on τ , we obtain the following expression for the distribution function:

$$\tau \int e^{ikr} f_{sk1}(\vec{\zeta}) dk = -\tau r^2 \left(\frac{\partial}{\partial \zeta_r} \int e^{ikr} f_{sk0}(\vec{\zeta}) dk \right) - \tau \zeta_r \int e^{ikr} (ik) f_{sk0}(\vec{\zeta}) dk, \quad (52)$$

that finally gives

$$f_{s1}(r, \vec{\zeta}) = -\Lambda r^2 \frac{\partial f_{s0}(r, \vec{\zeta})}{\partial \zeta_r} - \zeta_r \frac{\partial f_{s0}(r, \vec{\zeta})}{\partial r}. \quad (53)$$

Inserting expression Eq. (47) in Eq. (53), we obtain the first order distribution function term:

$$\begin{aligned} f_{s1}(r, \vec{\zeta}) &= -\zeta_r \frac{\partial f_0(r, \vec{\zeta})}{\partial r} - r^2 \Lambda \frac{\partial f_0(r, \vec{\zeta})}{\partial \zeta_r} - \zeta_r \Lambda_0 \frac{\partial K_1(\xi_0)}{\partial r} \frac{\partial f_0(r, \vec{\zeta})}{\partial \zeta_r} \\ &- \zeta_r \Lambda_0 K_1(\xi_0) \frac{\partial^2 f_0(r, \vec{\zeta})}{\partial \zeta_r \partial r} - r^2 \Lambda \Lambda_0 K_1(\xi_0) \frac{\partial^2 f_0(r, \vec{\zeta})}{\partial \zeta_r^2}. \end{aligned} \quad (54)$$

The validity of our perturbative expansion is controlled by the request that

$$r^2 \Lambda \tau < 1, \quad (55)$$

all other parameters in the Eq. (54) are small. Now we can estimate the maximal value of τ in the problems as

$$l_0 = cT \propto \hbar / mc, \quad (56)$$

see [18], and write $r^2 \Lambda$ with the help of Eq. (51) as

$$r^2 \Lambda \propto \frac{E_{pot}}{rmc^2} \quad (57)$$

with E_{pot} as a potential energy of the drop. Thereby, we can rewrite the inequality Eq. (55) as

$$E_{pot} < \frac{r}{l_0} mc^2. \quad (58)$$

Thereby, we see from Eq. (58), that the inequality Eq. (55) satisfies during all time of the applicability of Vlasov's equation, and, therefore, justify the expansion Eq. (46) in whole "time" of interest.

6 Transport properties of the drop

In this section, we calculate the averaged velocities and shear viscosity of the drop with the help of previously found distribution function. Namely, we will calculate the averaged radial velocity

$$\langle v_r \rangle = \frac{\int d^2 v v_r f_s}{\int d^2 v f_s}, \quad (59)$$

the averaged azimuthal flow velocity

$$\langle v_\theta \rangle = \frac{\int d^2 v v_\theta f_s}{\int d^2 v f_s}, \quad (60)$$

and the non-diagonal term of the stress-energy tensor required for the shear viscosity calculation

$$\sigma_{ij} = nm \int d^2 v (v_i - \langle v_i \rangle) (v_j - \langle v_j \rangle) f_s, \quad (61)$$

where $i, j = r, \theta$.

6.1 Radial velocity calculations

We calculate radial velocity up to the first order of perturbation series only, therefore we take everywhere a density profile of the drop equal to

$$N = \int d^2v f_s = \int d^2v (f_{s0} + \tau f_{s1}) = 1 - \Lambda_0 \frac{\partial K_1(\xi_0)}{\partial r} \int d^2v \zeta_r \frac{\partial f_0(r, \vec{\zeta})}{\partial \zeta_r} = 1 + \tau \Lambda_0 \frac{\partial K_1(\xi_0)}{\partial r}, \quad (62)$$

that gives for the first order term of the radial velocity

$$\langle v_r \rangle = N^{-1} \int d^2v v_r f_s = N^{-1} \left(\int d^2v v_r f_{s0} + \tau \int d^2v v_r f_{s1} \right) = N^{-1} \left(\langle v_r \rangle_s^0 + \tau \langle v_r \rangle_s^1 \right). \quad (63)$$

Here we used the lower subscript "s" as the sign of averaging over the f_s distribution function and the superscripts denote the orders of the term in the perturbation series.

The calculation of the first term in Eq. (63) is simple. We have:

$$\langle v_r \rangle_s^0 = \int d^2v v_r f_{s0} = \int d^2v v_r \left(f_0(r, \vec{\zeta}) + \Lambda_0 K_1(\xi_0) \frac{\partial f_0(r, \vec{\zeta})}{\partial \zeta_r} \right). \quad (64)$$

Opening the brackets we obtain:

$$\langle v_r \rangle_s^0 = \langle v_r \rangle_0 + c \Lambda_0 K_1(\xi_0) \int d^2v v_r \frac{\partial f_0(r, \vec{v})}{\partial v_r}. \quad (65)$$

Taking into account the properties of our initial distribution f_0 , we obtain finally:

$$\langle v_r \rangle_s^0 = -c \Lambda_0 K_1(\xi_0) \int d^2v f_0(r, \vec{v}) = -c \Lambda_0 K_1(\xi_0). \quad (66)$$

We see, that as we assumed for the $\Lambda_0 \ll 1$, our radial velocity is small $\langle v_r \rangle_s^0 \ll c$. We obtained also, that the value of this velocity fully determined by the influence of the external field, and contrary to the results of Appendix A, the radial velocity is not zero even in zero order.

In the first order of perturbation we have:

$$\langle v_r \rangle_s^1 = \int d^2v v_r f_{s1} = - \int d^2v v_r \left(\Lambda r^2 \frac{\partial f_{s0}}{\partial \zeta_r} + \zeta_r \frac{\partial f_{s0}}{\partial r} \right). \quad (67)$$

Taking into account the full answer for the distribution function in this order Eq. (54), we can rewrite Eq. (67) as

$$\langle v_r \rangle_s^1 = \int d^2v v_r f_{s1} = - \int d^2v v_r \left(\Lambda r^2 \frac{\partial f_0}{\partial \zeta_r} + \zeta_r \frac{\partial f_0}{\partial r} \right), \quad (68)$$

all other terms gives zero contribution to the radial velocity value in this order.

The first term in the r.h.s. in Eq. (68) is

$$\Lambda r^2 \int d^2v v_r \frac{\partial f_0}{\partial \zeta_r} = c \Lambda r^2 \int d^2v v_r \frac{\partial f_0}{\partial v_r} = -c \Lambda r^2 \int d^2v f_0 = -c \Lambda r^2, \quad (69)$$

whereas the second term is

$$\int d^2v v_r \zeta_r \frac{\partial f_0}{\partial r} = \frac{1}{c} \frac{\partial}{\partial r} \int d^2v v_r^2 f_0 = \frac{1}{c} \frac{\partial \langle v_r^2 \rangle_0}{\partial r}. \quad (70)$$

Here we need to calculate the square of the radial velocity averaged over Eq. (A.1) distribution function:

$$\langle v_r^2 \rangle_0 = \int d^2v v_r^2 f_0. \quad (71)$$

We have:

$$\begin{aligned} 2 \langle v_r^2 \rangle_0 &= 2 \left(\frac{m}{2\pi} \right) \int d^2v v_r^2 \delta \left(\frac{1}{2m} (p_r^2 + (p_\theta - m\omega_r r)^2) + \psi(r) - kT_\perp \right) \\ &= \left(\frac{m}{2\pi} \right) \int d^2v (v_r^2 + v_\theta^2) \delta \left(\frac{1}{2m} (p_r^2 + p_\theta^2) + \psi(r) - kT_\perp \right), \end{aligned} \quad (72)$$

defining a new variable:

$$u^2 = \frac{1}{2m} (p_r^2 + p_\theta^2), \quad (73)$$

after the variable change we get the following expression:

$$2 \langle v_r^2 \rangle_0 = \left(\frac{2}{m} \right) \int du^2 u^2 \delta(u^2 + \psi(r) - k T_\perp), \quad (74)$$

which gives with the help of Eq. (A.16):

$$\langle v_r^2 \rangle_0 = \left(\frac{1}{m} \right) (k T_\perp - \psi(r)) = \frac{k T_\perp}{m} \left(1 - \frac{r^2}{r_b^2} \right). \quad (75)$$

Taking both terms together, we obtain:

$$\langle v_r \rangle_s^1 = c \left(\frac{2 k T_\perp r}{m c^2 r_b^2} + \frac{2 \pi q^2 n r}{m c^2} \right) = \frac{c r}{r_b^2} \left(\frac{2 k T_\perp}{m c^2} + \frac{2 \pi q^2 n r_b^2}{m c^2} \right). \quad (76)$$

We see, that we have here two possible different regimes. The first one for the weakly interacting plasma, when

$$\Gamma = \frac{|\hat{U}_p|}{k T_\perp} \ll 1 \quad (77)$$

we obtain

$$\langle v_r \rangle_s^1 \approx c \frac{2 k T_\perp r}{m c^2 r_b^2} \quad (78)$$

whereas for the strongly interacting plasma when

$$\Gamma \gg 1 \quad (79)$$

we obtain

$$\langle v_r \rangle_s^1 \approx c \frac{2 \pi q^2 n r}{m c^2}. \quad (80)$$

In both cases, the corrections are small due the large value of $m c^2$ in denominators of Eq. (78)-Eq. (80).

The final expression for radial velocity up to the first order, therefore, is the following:

$$\langle v_r \rangle = -N^{-1} \left(c \Lambda_0 K_1(\xi_0) + \tau \frac{c r}{r_b^2} \left(\frac{2 k T_\perp}{m c^2} + \frac{2 \pi q^2 n r_b^2}{m c^2} \right) \right). \quad (81)$$

Taking

$$N^{-1} \approx 1 - \tau \Lambda_0 \frac{\partial K_1(\xi_0)}{\partial r} \quad (82)$$

and keeping in final answer only liner on τ terms we obtain:

$$\langle v_r \rangle = -c \Lambda_0 \left(K_1(\xi_0) - \frac{\tau \Lambda_0}{r_{0D}} K_1(\xi_0) \frac{\partial K_1(\xi_0)}{\partial \xi_0} \right) + \tau \frac{c r}{r_b^2} \left(\frac{2 k T_\perp}{m c^2} + \frac{2 \pi q^2 n r_b^2}{m c^2} \right). \quad (83)$$

We see, that whereas the first term of expression Eq. (81) is fully determined by the external field and negative (compression process), the first order perturbation term has positive sign and describes the instability of the charged droplet in absence of permanent external electric field. Further, equation Eq. (67) we will use in the form similar to Eq. (63):

$$\langle v_r \rangle = \langle v_r \rangle_s^0 + \tau \langle v_r \rangle_s^1, \quad (84)$$

where we group all liner on τ terms in the $\langle v_r \rangle_s^1$ term.

6.2 Azimuthal velocity calculations

We calculate the azimuthal velocity up to the first order similarly to the radial velocity calculations:

$$\langle v_\theta \rangle = N^{-1} \int d^2v v_\theta f_s = N^{-1} \left(\int d^2v v_\theta f_{s0} + \tau \int d^2v v_\theta f_{s1} \right) = N^{-1} \left(\langle v_\theta \rangle_s^0 + \tau \langle v_\theta \rangle_s^1 \right). \quad (85)$$

In the first order we have:

$$\langle v_\theta \rangle_s^0 = \int d^2v v_\theta f_{s0} = \int d^2v v_\theta \left(f_0(r, \vec{\zeta}) + \Lambda_0 K_1(\xi_0) \frac{\partial f_0(r, \vec{\zeta})}{\partial \zeta_r} \right) = \int d^2v v_\theta f_0(r, \vec{\zeta}). \quad (86)$$

We see that in this order, our answer is simply Eq. (A.7):

$$\langle v_\theta \rangle_s^0 = \langle v_\theta \rangle_0 = \omega_r r. \quad (87)$$

There is no influence of the external field in this order.

In the next order, taking into account the first perturbed term, we have:

$$\langle v_\theta \rangle_s^1 = \int d^2v v_\theta f_{s1} = - \int d^2v v_\theta \zeta_r \Lambda_0 K_1(\xi_0) \frac{\partial^2 f_0(r, \vec{\zeta})}{\partial \zeta_r \partial r}. \quad (88)$$

All other terms of Eq. (54) give zero contribution to the integral Eq. (67). Thereby we have:

$$\langle v_\theta \rangle_s^1 = -\Lambda_0 K_1 \int d^2v v_\theta v_r \frac{\partial^2 f_0}{\partial v_r \partial r} = \Lambda_0 K_1 \frac{\partial}{\partial r} \int d^2v v_\theta f_0 = \Lambda_0 K_1 \frac{\partial \langle v_\theta \rangle_0}{\partial r}. \quad (89)$$

Taking both terms of Eq. (67)-Eq. (68) together and accounting Eq. (82) we obtain:

$$\langle v_\theta \rangle = \omega_r r \left(1 - \tau \Lambda_0 \frac{\partial K_1(\xi_0)}{\partial r} \right) + \tau \Lambda_0 K_1 \omega_r. \quad (90)$$

In this case the external perturbation leads to the increase of the droplet's rotation velocity. We also rewrite this expression in more useful form. Collecting all liner on τ terms together we rewrite Eq. (90) as

$$\langle v_\theta \rangle = \langle v_\theta \rangle_s^0 + \tau \langle v_\theta \rangle_s^1. \quad (91)$$

This form of the answer we will use in the next subsection.

6.3 Transverse shear viscosity of the drop

The shear viscosity is defined by the non-diagonal term of the stress-energy tensor. We have:

$$\sigma_{r\theta} = n m \int d^2v (v_\theta - \langle v_\theta \rangle) (v_r - \langle v_r \rangle) f_s. \quad (92)$$

In the zero order of our perturbation series, we have then

$$\sigma_{r\theta}^0 = n m \int d^2v (v_\theta v_r - v_r \langle v_\theta \rangle_s^0 - v_\theta \langle v_r \rangle_s^0 + \langle v_r \rangle_s^0 \langle v_\theta \rangle_s^0) f_{s0} = 0, \quad (93)$$

as it must be in the case of equilibrium. In the first order of perturbations we have:

$$\begin{aligned} \sigma_{r\theta}^0 + \sigma_{r\theta}^1 &= n m \int d^2v (v_\theta v_r - v_r (\langle v_\theta \rangle_s^0 + \tau \langle v_\theta \rangle_s^1) - \\ &- v_\theta (\langle v_r \rangle_s^0 + \tau \langle v_r \rangle_s^1) + (\langle v_r \rangle_s^0 + \tau \langle v_r \rangle_s^1) (\langle v_\theta \rangle_s^0 + \tau \langle v_\theta \rangle_s^1)) (f_{s0} + \tau f_{s1}). \end{aligned} \quad (94)$$

So we obtain:

$$\sigma_{r\theta}^1 = n \tau \left(\langle v_r v_\theta \rangle_s^1 - \langle v_r \rangle_s^0 \langle v_\theta \rangle_s^1 - \langle v_r \rangle_s^1 \langle v_\theta \rangle_s^0 + \langle v_r \rangle_s^0 \langle v_\theta \rangle_s^0 \int d^2v f_{s1} \right). \quad (95)$$

The only unknown term in the r.h.s. of the equation is the following one:

$$\langle v_r v_\theta \rangle_s^1 = \int d^2 v v_\theta v_r f_{s1}, \quad (96)$$

with the f_{s1} is given by Eq. (54). Keeping only non-vanishing after the integration terms we obtain:

$$\langle v_r v_\theta \rangle_s^1 = - \int d^2 v v_\theta v_r \left(\zeta_r \frac{\partial f_0(r, \vec{\zeta})}{\partial r} + r^2 \Lambda \frac{\partial f_0(r, \vec{\zeta})}{\partial \zeta_r} \right). \quad (97)$$

Integration of this expression gives, for the first integrand:

$$- \int d^2 v v_\theta v_r \zeta_r \frac{\partial f_0(r, \vec{\zeta})}{\partial r} = - \frac{\langle v_\theta \rangle_s^0}{c} \frac{\partial}{\partial r} \int d^2 v v_r^2 f_0 = - \frac{\langle v_\theta \rangle_s^0}{c} \frac{\partial \langle v_r^2 \rangle_0}{\partial r}, \quad (98)$$

and for the second one:

$$- \int d^2 v v_\theta v_r r^2 \Lambda \frac{\partial f_0(r, \vec{\zeta})}{\partial \zeta_r} = r^2 \Lambda c \int d^2 v v_\theta f_0 = r^2 \Lambda c \langle v_\theta \rangle_s^0. \quad (99)$$

Taking into account Eq. (67)-Eq. (76), Eq. (86)-Eq. (89) and summing up all terms we obtain :

$$\sigma_{r\theta}^1 = \eta_{r\theta}^{\sigma\delta} \frac{\partial \langle v_\sigma \rangle_s^0}{\partial r_\delta} = n m \tau c (\Lambda_0 K_1)^2 \frac{\partial \langle v_\theta \rangle_s^0}{\partial r} + 3 n m \tau \omega_r r \Lambda_0 K_1 \frac{\partial \langle v_r \rangle_s^0}{\partial r}. \quad (100)$$

This expression determines the shear viscosity coefficients in transverse plane:

$$\eta_{r\theta}^{\theta r} = n m \tau c (\Lambda_0 K_1)^2, \quad \eta_{r\theta}^{rr} = 3 n m \tau \omega_r r \Lambda_0 K_1. \quad (101)$$

This result is interesting. Both coefficients are proportional to the external field determined by the Λ_0 parameter which is small. Also, these coefficients are proportional to the τ and, therefore, stays very small during evolution of the system if τ remains small. Similarly to the definition in [21] we can call this viscosity coefficients as anomalous.

Indeed, the parameter τ in Eq. (101) is not a mean free path as in usual interpretations of the viscosity in different kinematic approximations. The shear viscosity coefficients of Eq. (101) are determined by the collective expansion of the charged particles under an influence of the interactions of the particles inside the droplet and some initial perturbation caused by an external field. Thereby, the expressions for this anomalous viscosity values are correct till the Vlasov's approximation is correct in the description of the expanding dense matter. Therefore, the τ parameter in Eq. (101) is a "time" of evolution of the droplet in the phase of the hydrodynamic expansion (compression) and the maximum value of this parameter is itself small. We estimate the value of τ approximately as:

$$\tau \propto \hbar / m c, \quad (102)$$

see for example [18]. Inserting Eq. (102) into Eq. (101) we obtain:

$$\eta_{r\theta}^{\theta r} \rightarrow \hbar n (\Lambda_0 K_1)^2 \approx \hbar n \left(\frac{E_p^{ext}}{\mathcal{E}_{kin}} \right)^2 \quad (103)$$

and

$$\eta_{r\theta}^{rr} \rightarrow 2 \hbar n \frac{\omega_r r}{c} \frac{E_p^{ext}}{\mathcal{E}_{kin}}, \quad (104)$$

where we took $K_1 \propto 1$ for all r of interest. We see, that the obtained coefficients change from zero value till very small maximum value during the droplet's expansion. Indeed, the only large parameter in Eq. (103) is the density of the droplet n , which can be very large, but Λ_0 is a small parameter of the approach and overall value of the viscosity is proportional to \hbar . Additionally, in the case when $\omega_r = 0$ the second coefficient is zero as well.

Obviously, the viscosity/entropy ratio in our calculations remains small. Indeed, we consider Vlasov equation, the entropy during the process of the drop's expansion remains constant and it equals to the initial entropy of the drop: $s = s_0 = const^5$. Therefore, the ratio $\eta/s = \eta/s_0$ changes only because change of the viscosity coefficient η and overall ratio changes from zero to some small value determined by the viscosity coefficient value Eq. (103)-Eq. (104).

⁵Here s is an entropy of the process

7 Conclusion

In this paper, we considered a process of the compression/expansion of the dense charged droplet in the transverse plane under the influence of the external transverse electric field. The main motivation of our calculations was the investigation of the possibility of shear viscosity to entropy ratio smallness in the framework of some classical model. We considered a case of non-stationary transverse drop expansion/compression in the hydrodynamical phase with constant entropy. The main result of our calculations is given by Eq. (101)-Eq. (104) for the shear viscosity coefficients value, which really provides requested smallness.

The droplet we considered, consists of charged particles, the external fields are not constant in time, therefore the droplet cannot stay in the equilibrium state and begins to expand shortly after it's creation. The very early stage of the expansion of the droplet is hydrodynamical one, [18], the collisions between particles are not important and the expansion happens with constant entropy. Thereby, the non-equilibrium, time-dependent distribution function of the droplet's evolution is determined by Vlasov's equation with the external field included.

We solve a classical problem for the distribution function of the non-equilibrium system, therefore, first of all, it is instructive to check the self-consistency of the obtained solution. Beginning from the "rigid body" initial state, Appendix A, we can "guess" the influence of the external electric field on the droplet and compare it with the obtained answers. In the framework of the definitions of the paper, we have that the case $\Lambda_0 > 0$ corresponds to the repulsive external force, that means droplet's external compression, and the case $\Lambda_0 < 0$ corresponds to the attractive external force, that means droplet's external expansion, the case $\Lambda_0 = 0$ corresponds to the absence of the interaction. In our case we consider electromagnetic interactions, but in general, other types of interactions between the droplets are possible in more complex models, see for example [27, 28].

This processes of external compression/expansion are applied to the process of droplet's expansion under the influence of charged particles inside the drop. From this point of view we see that our results are self-consistent and coincide with the naive expectations from the process. Indeed, the radial velocity answer, Eq. (81), consists of the two terms. The first one is due to the external field, depending on the sign of Λ_0 which can be negative, case of the compression, or positive, case of the expansion. The second term in Eq. (81) is positive and corresponds to the expansion of the droplet under the influence of the repulsive forces between charged particles in the drop. The azimuthal velocity expression, Eq. (90), also consist of the two terms. The first one is the same as obtained for the initial distribution in Appendix A, while the second one is the correction to the initial value due to the external field. We see here, that for the case of the compression, the azimuthal velocity increases, and in the case of negative Λ_0 it decreases, as it must be according with the classical point of view. The density profile of the drop does not remains constant as well. In the first order on τ it changes under an influence of the external field. The instability of the charged particles inside the drop influences only in the next orders on τ .

It was already mentioned above, that the important result of the performed calculations is the expression for the shear viscosity coefficient Eq. (101). It would be underlined, that the value of the coefficient is determined by the interaction of the droplet with the external field, i.e. due to the non-zero Λ_0 coefficient which depends on the strength of the interactions between charged droplets⁶. Also, the "time" parameter in Eq. (101) is the evolution "time" of the process, not the mean free path, and it's value is limited by the value of the "time's" applicability of the Vlasov's approximation, Eq. (102), which itself is small. Thereby, our viscosity is time dependent and changing during the drop's expansion, but it remains very small, also because Λ_0 determined by Eq. (41) is a perturbative, small parameter of the initial conditions. This mechanism of the viscosity coefficients smallness can be called as an anomalous one, similarly to [21]. The entropy of the process remains constant during the process of the drop's expansion/compression. It gives immediately that the ratio viscosity/entropy changes from zero to some value during the process of interest, but due to the smallness of the viscosity, this ratio anyway remains very small, see also [29].

In our model we considered a droplet, which we assume to be similar to the drops which are created in the very initial stage of the high-energy scattering of hadrons. Initially these drops are in the state

⁶In the case when $\Lambda_0 = 0$ the viscosity should be determined by the next order in the τ perturbative expansion of the distribution function and transport coefficients.

of the thermal equilibrium and the smallness of the drop's size is required in this case, see [25]. This required smallness of the drops leads to simple consequences which concern our calculations. In QCD such small drop will contain almost asymptotically free quarks and gluons, which interactions in this phase in some extent are similar to the electromagnetic interactions. Thereby, we can learn important lessons about QGP behavior even from the present, oversimplified calculations. Namely, if we assume that drops similar to considered are created in high energy collisions, then the transverse dynamics of the drops must be similar to the obtained here with only interaction coupling values different. We see therefore, that the smallness of the shear viscosity to entropy ratio may be explained even in the framework of classical models for the systems of weakly interacting particles.

In order to make the calculations clearer, we limited the consideration of the droplet's dynamic by the transverse plane only. In this case, the longitudinal dynamics is absent and, as was shown below, magnetic fields may be neglected. Definitely, in this approximation, our results can be used in the case of three dimensional dynamics with some approximation only. However, the obtained form of Vlasov's equation Eq. (33) is linear over the magnetic and electric fields and they are linearly independent in the expression. Therefore, the inclusion of magnetic field in the model will definitely change the value of the radial force in the final answers but we doubt that it will change also the properties and general structures of the obtained expressions for the transverse plane. Some additional terms will be added to the answers after all. Our future calculations will include the magnetic field as well as the longitudinal dynamics, the present model we consider as a very preliminary step towards full dynamics description of the processes of interest.

In general we conclude thereby, that we performed the first step in the description of the evolution of dense droplets in the presence of external electromagnetic fields. This task is important because in some extent, our model can clarify a dynamics of interactions of quarks and gluons in the colored charged drops in QCD processes and mechanisms of shear viscosity smallness in it. Also, we hope, that the microscopical details of the hydrodynamical expansion of the charged drops may provide better understanding and description of the data obtained in high-energy collisions of protons and nuclei in the LHC and RHIC experiment, [10],[29].

Appendix A: A "rigid-rotor" initial equilibrium distribution function

Properties of initial distribution in our problem are defined by Eq. (38), which, in turn, depends on some another function $f_0(r, \vec{p})$. As $f_0(r, \vec{p})$ we will choose a "rigid-rotor" equilibrium function:

$$f_0(r, \vec{p}) = \left(\frac{m}{2\pi} \right) \delta(H_\perp - \omega_r P_\theta - k T_\perp) G(p_z), \quad (\text{A.1})$$

where we have:

$$\int_{-\infty}^{\infty} G(p_z) dp_z = 1; \quad (\text{A.2})$$

$$\left(\frac{m}{2\pi} \int_{-\infty}^{\infty} dv_\theta \int_{-\infty}^{\infty} dv_r \delta(H_\perp - \omega_r P_\theta - k T_\perp) \right)_{r=0} = 1. \quad (\text{A.3})$$

the non-relativistic Hamiltonian of the problem is given by

$$H_\perp = \frac{1}{2m} (p_r^2 + p_\theta^2) + q \Phi_{s0}, \quad (\text{A.4})$$

where in Eq. (A.1)

$$P_\theta = r (p_\theta - m \omega_c r / 2), \quad (\text{A.5})$$

and

$$\omega_c = \frac{|q| B_0}{m c} \quad (\text{A.6})$$

is a cyclotron frequency in presence of some initial magnetic field in z direction, see [32]. An average azimuthal flow velocity

$$\langle v_\theta \rangle_0 = \frac{\int d^3p v_\theta f_0}{\int d^3p f_0} = \omega_r r \quad (\text{A.7})$$

is fully defined by some given angular velocity $\omega_r = \text{const}$ that explains the name "rigid-rotor" equilibrium for the Eq. (A.1) function.

It is convenient to rewrite argument of delta function in Eq. (A.3) in the following form:

$$H_\perp - \omega_r P_\theta = \frac{1}{2m} (p_r^2 + (p_\theta - m \omega_r r)^2) + \psi(r), \quad (\text{A.8})$$

where an effective potential $\psi(r)$ is defined as

$$\psi(r) = \frac{m}{2} (\omega_c \omega_r - \omega_r^2) r^2 + q \Phi_{s0}. \quad (\text{A.9})$$

The corresponding Poisson equation for the self-consistent potential Φ_{s0} can be solved in this case giving

$$\Phi_{s0} = -\pi n q r^2 \quad (\text{A.10})$$

and we can rewrite potential Eq. (A.9) as

$$\psi(r) = \frac{m}{2} (\omega_r^+ - \omega_r) (\omega_r - \omega_r^-) r^2 \quad (\text{A.11})$$

with

$$\omega_r^\pm = \frac{\omega_c}{2} \{1 \pm (1 - n s_q)^{1/2}\} \quad (\text{A.12})$$

and

$$s_q = \frac{8\pi q^2}{m \omega_c^2} \quad (\text{A.13})$$

Now we see that for the distribution function Eq. (A.1) the density profile is constant:

$$n(r) = \begin{cases} 1 = \text{const.}, & 0 \leq r < r_b \\ 0, & r > r_b. \end{cases} \quad (\text{A.14})$$

with

$$r_b^2 = \frac{2k T_\perp / m}{(\omega_r^+ - \omega_r)(\omega_r - \omega_r^-)} \quad (\text{A.15})$$

as some maximal radius of non-zero density. With the help of Eq. (A.15), Eq. (A.11) acquires the following form:

$$\psi(r) = \frac{k T_{\perp} r^2}{r_b^2}. \quad (\text{A.16})$$

In the following we consider the case when

$$r_D \leq r_b, \quad (\text{A.17})$$

that makes approximation Eq. (A.11) correct in the region of interest of our problem.

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